



Boğaziçi University

Introductory Phys Labs

THE PHYSICAL PENDULUM

PHYS 102-130

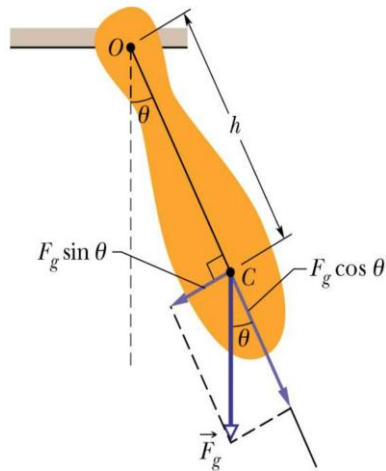
IMPORTANT INSTRUCTIONS

- Use a computer to watch the Lab videos.
- All data in the presentations are only used for demonstrational purposes. DO NOT use the data from presentations.
- Before each lab session starts make sure that you print the pages involving the relevant experiment of the Lab textbook whose copy can be found in the lab web page (<https://physlab.boun.edu.tr>).
- You are responsible from this experiment in the lab final exam. Study the presentation, fill your lab-sheets using the data video and fill the post-lab sheet and sign it. We will collect them in the following week.
- DO NOT FORGET TO BRING THE POST-LAB SHEET TO THE NEXT LAB SESSION.

THEORY

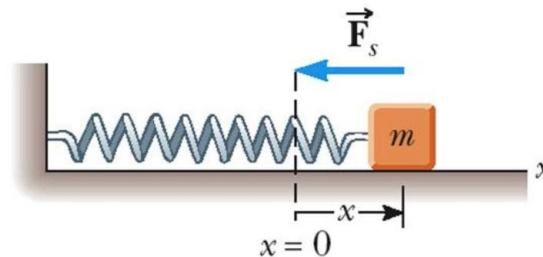
Simple harmonic motion is a special type of periodic motion where the restoring force on the moving object is directly proportional to the object's displacement magnitude and acts towards the object's equilibrium position.

Physical Pendulum



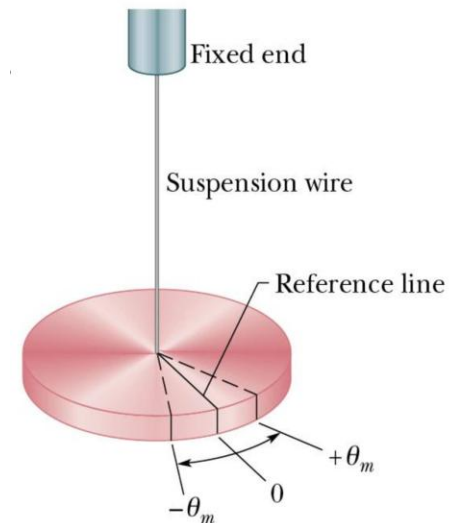
$$T = 2\pi \sqrt{\frac{I}{Mgh}}$$

Simple Harmonic Oscillator



$$T = 2\pi \sqrt{\frac{m}{k}}$$

Torsional Pendulum



$$T = 2\pi \sqrt{\frac{I}{\kappa}}$$

A **physical pendulum**, is any object whose oscillations are similar to those of the simple pendulum, but **cannot be modeled as a point mass** on a string, and mass distribution must be included into the equation of motion.

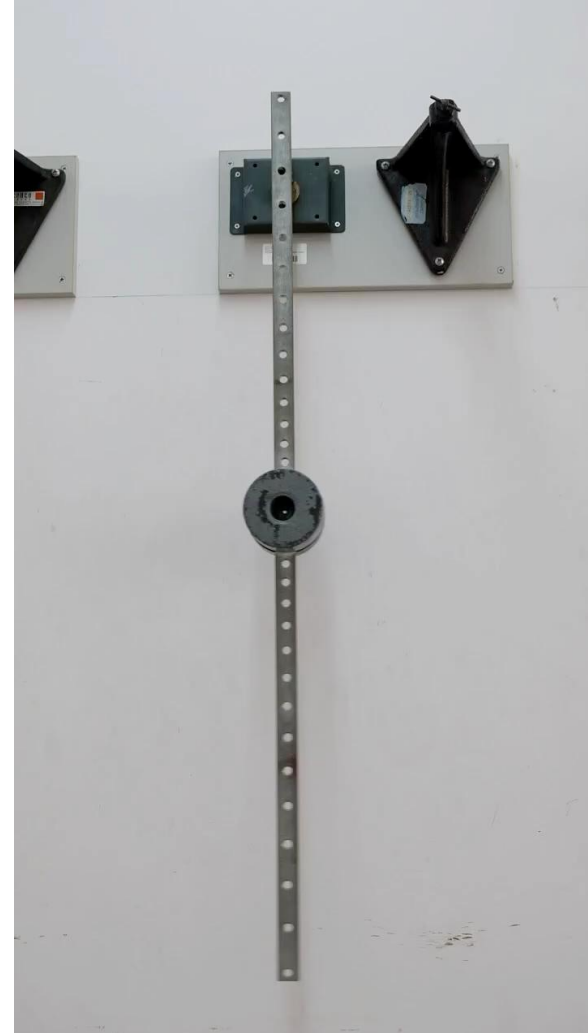


THE PHYSICAL PENDULUM

As for simple pendulum, the restoring force of the physical pendulum is the force of gravity. In the simple pendulum, the force of gravity acts on the center of the pendulum bob. In the case of physical pendulum, the force of gravity acts on the center of mass (CM) of an object.



- ❖ A physical pendulum makes simple harmonic motion if the amplitude is small.
- ❖ Gravity acts on the center of mass to rotate the pendulum. The pendulum continues to swing back and forth.

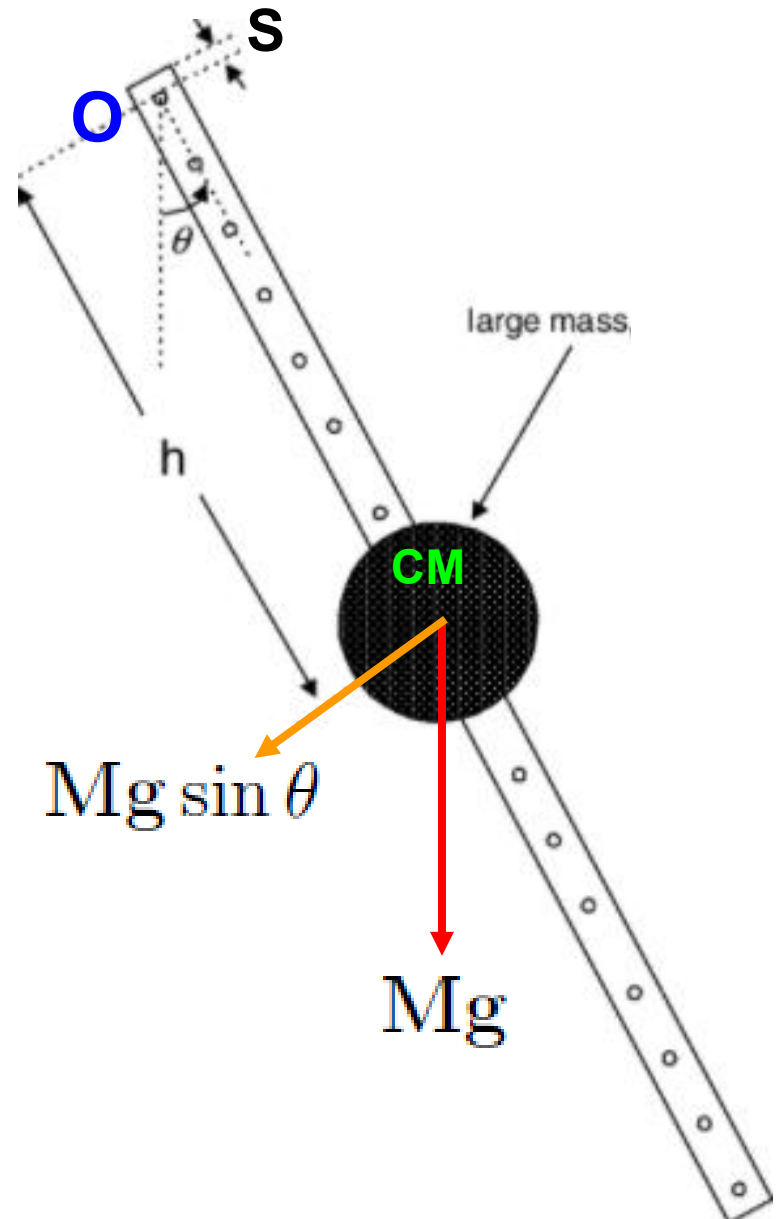


THE PHYSICAL PENDULUM

Consider a rigid body that can freely rotate around a horizontal axis through a fixed center of suspension O .

The gravitational force produces a torque about the suspension point O having magnitude:

$$\tau = Mgh \sin \theta$$



THE PHYSICAL PENDULUM

- Let us use the rotational form of Newton's second law : $\tau = I \alpha$

(I is the moment of inertia about the axis through pivot point O)

(α : angular acceleration)

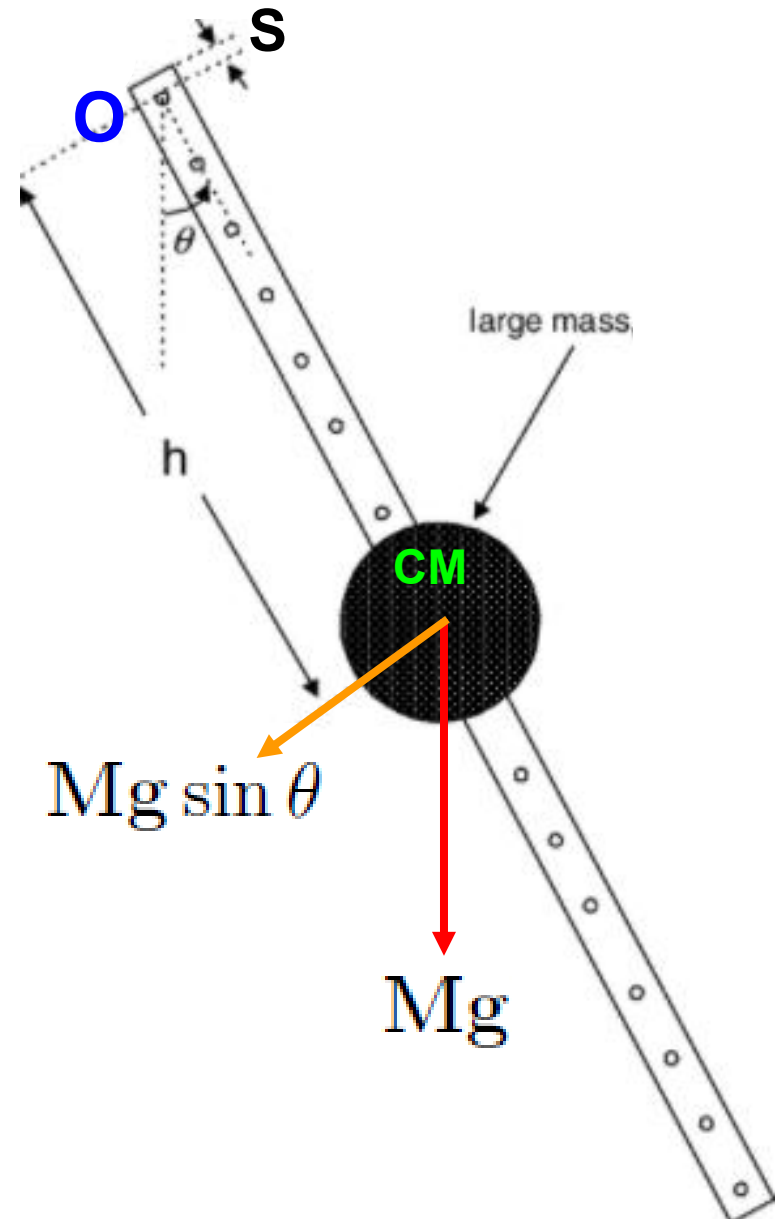
$$\alpha = \frac{d^2\theta}{dt^2}$$

Equating two torque expressions gives

$$I \frac{d^2\theta}{dt^2} = -Mgh \sin \theta$$

Now, if we assume θ is small, we have

$$\sin \theta \approx \theta$$



Finally, the equation of motion of the rigid body becomes

$$\frac{d^2\theta}{dt^2} + \frac{Mgh}{I}\theta = 0$$

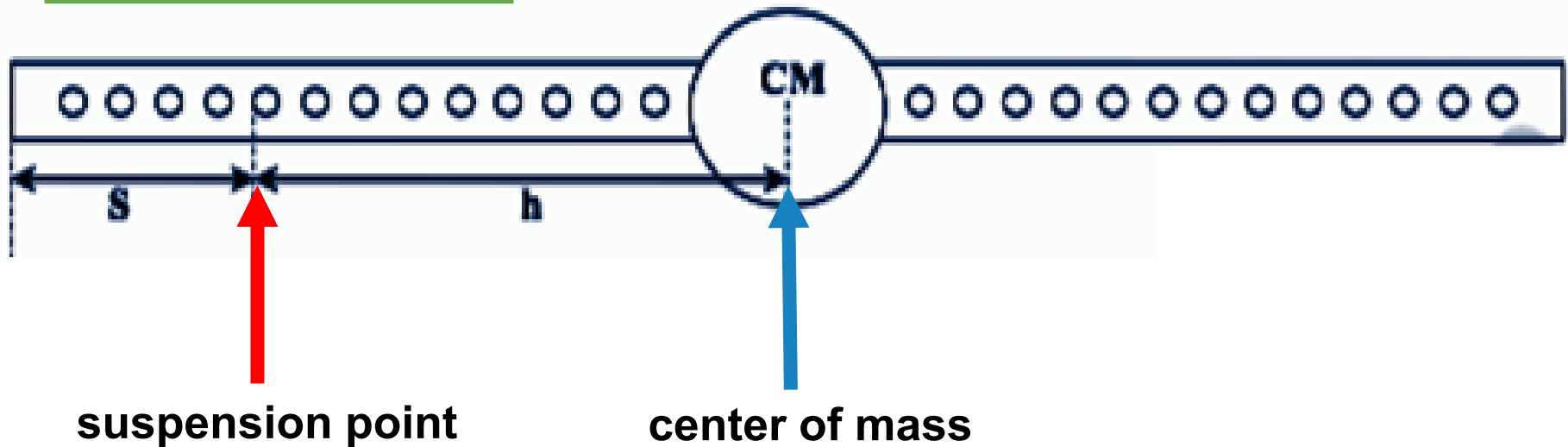
- This is in the form of the equation of **Simple Harmonic Motion**.
- The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$


$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

$$T = \frac{2\pi}{\omega}$$

Moment Of Inertia



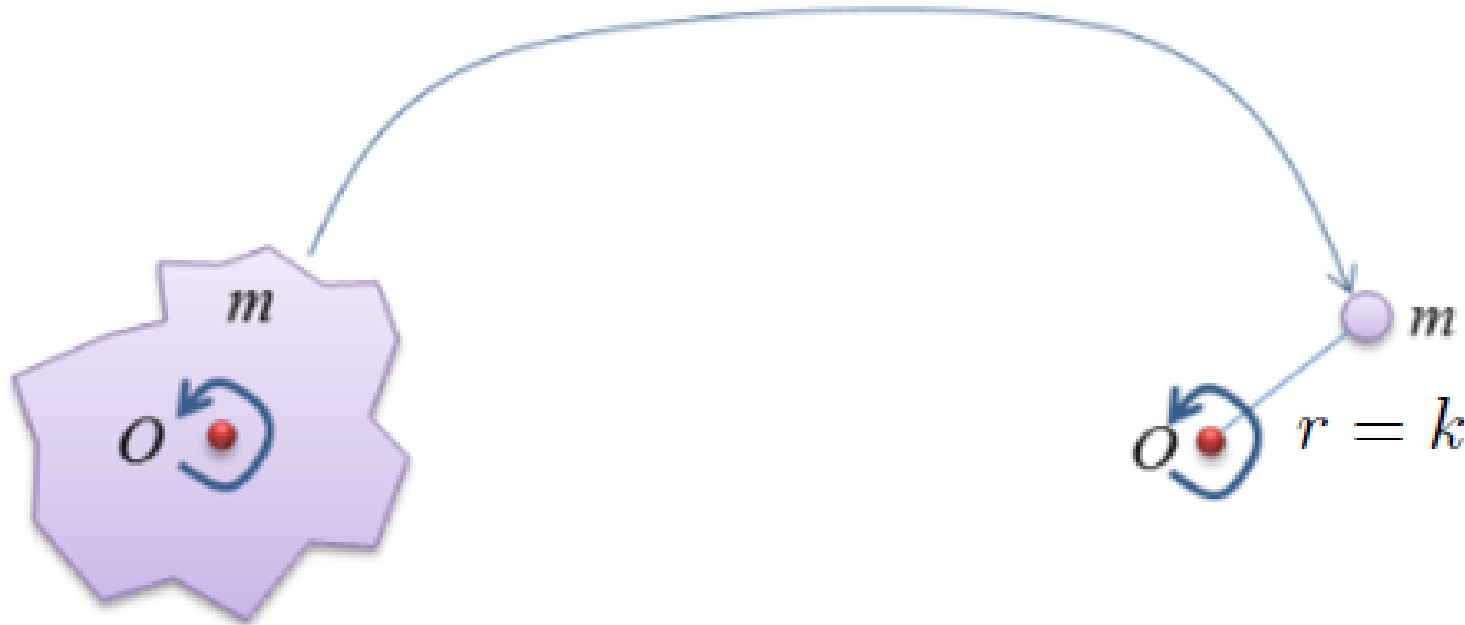
The moment of inertia of a rigid body about any given point can be expressed in terms of the moment of inertia about the center of mass using the parallel axis theorem:

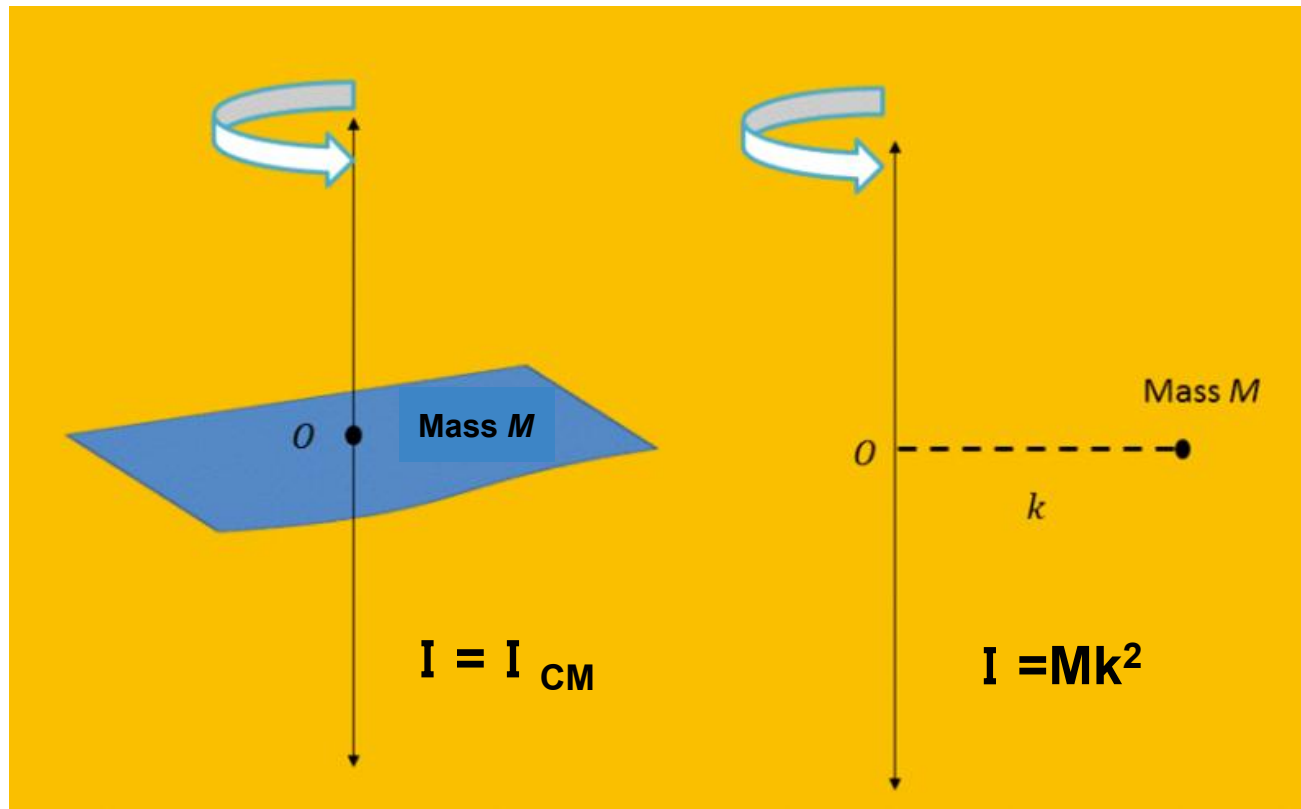
$$I = I_{CM} + Mh^2$$

So, what is the moment of inertia about the center of mass?

Radius Of Gyration (k) :

The radius of gyration k is the radial distance from the axis of rotation at which the total mass of a rigid body is assumed to be concentrated while the moment of inertia of the body remains the same.

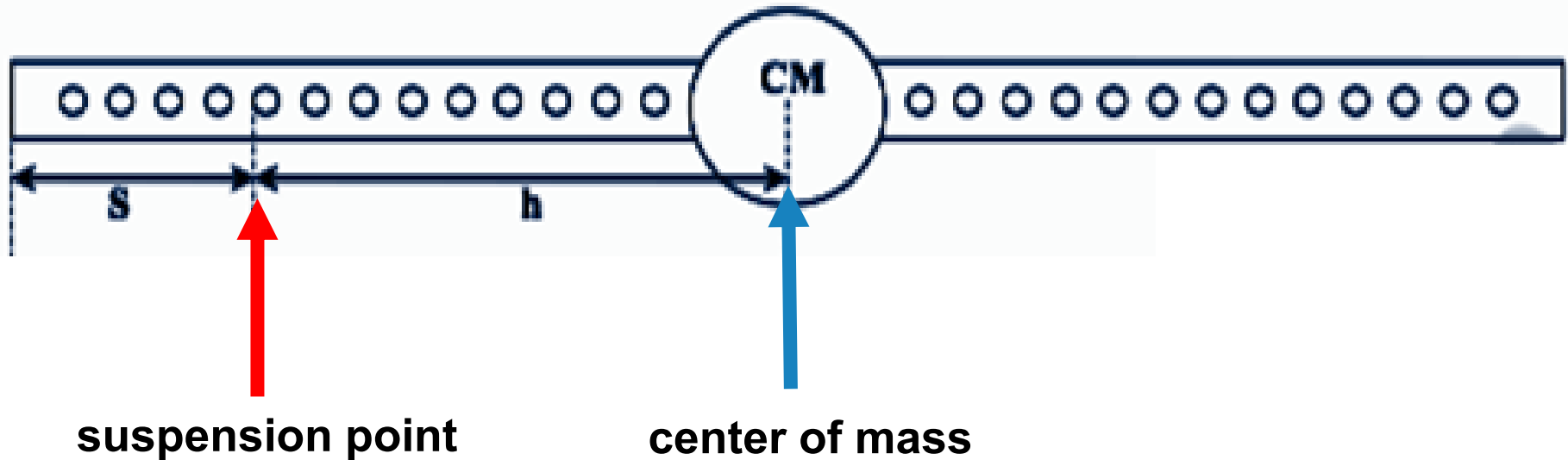




For a rigid body, the moment of inertia about center of mass can be written in terms of radius of gyration k :

$$I_{CM} = Mk^2$$

THE PHYSICAL PENDULUM

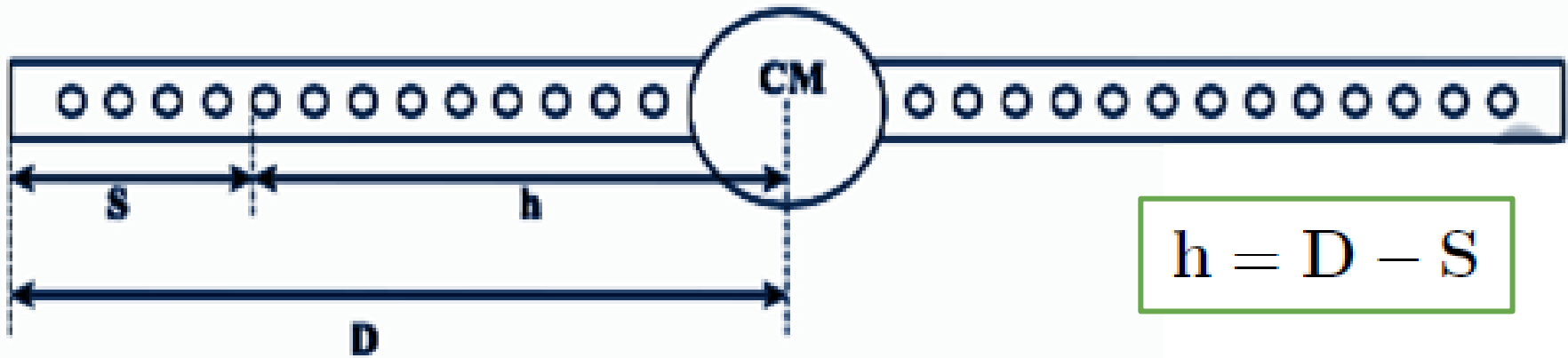


Then, the moment of inertia about the suspension point at a distance h from the center of mass is

$$I = M(k^2 + h^2)$$

$$I = I_{CM} + Mh^2$$

THE PHYSICAL PENDULUM



$$h = D - S$$

Period of Physical Pendulum : Recall that

$$T = 2\pi \sqrt{\frac{I}{Mgh}}$$

Combining the period expression with the moment of inertia, we can express the period as

$$T = 2\pi \sqrt{\frac{k^2 + h^2}{gh}}$$

The period of simple pendulum :



$$T = 2\pi\sqrt{\frac{L}{g}}$$

Period of Physical Pendulum :

$$T = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$$

★ The physical pendulum is **equivalent to a simple pendulum** with length:



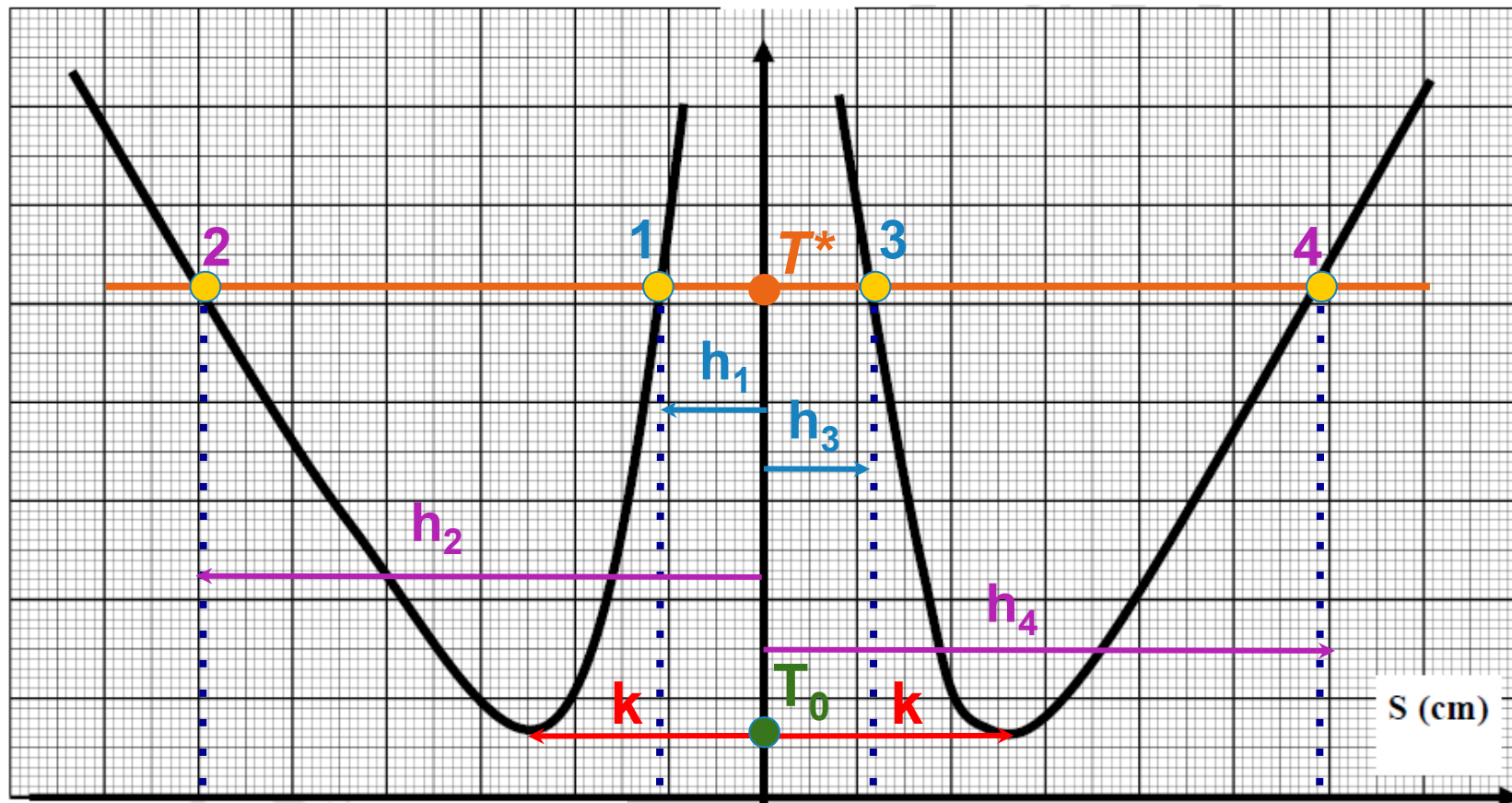
$$L = \frac{k^2 + h^2}{h}$$

THE PHYSICAL PENDULUM

If we plot T vs. S , we see that there are four possible points for a specific period value T^* that we can hang the pendulum.

→ Radius of gyration k gives the minimum period ($h=0$ case).

T (sec)



D

THE PHYSICAL PENDULUM

For one side of the pendulum, two different distances h_1 and h_2 from the center of mass give the same period:

$$2\pi\sqrt{\frac{k^2 + h_1^2}{gh_1}} = 2\pi\sqrt{\frac{k^2 + h_2^2}{gh_2}}$$

Solving this equation for the radius of gyration yields

$$k^2 = h_1 h_2$$

Period

$$T = 2\pi\sqrt{\frac{h_1 + h_2}{g}}$$

Length of Equivalent
Simple Pendulum

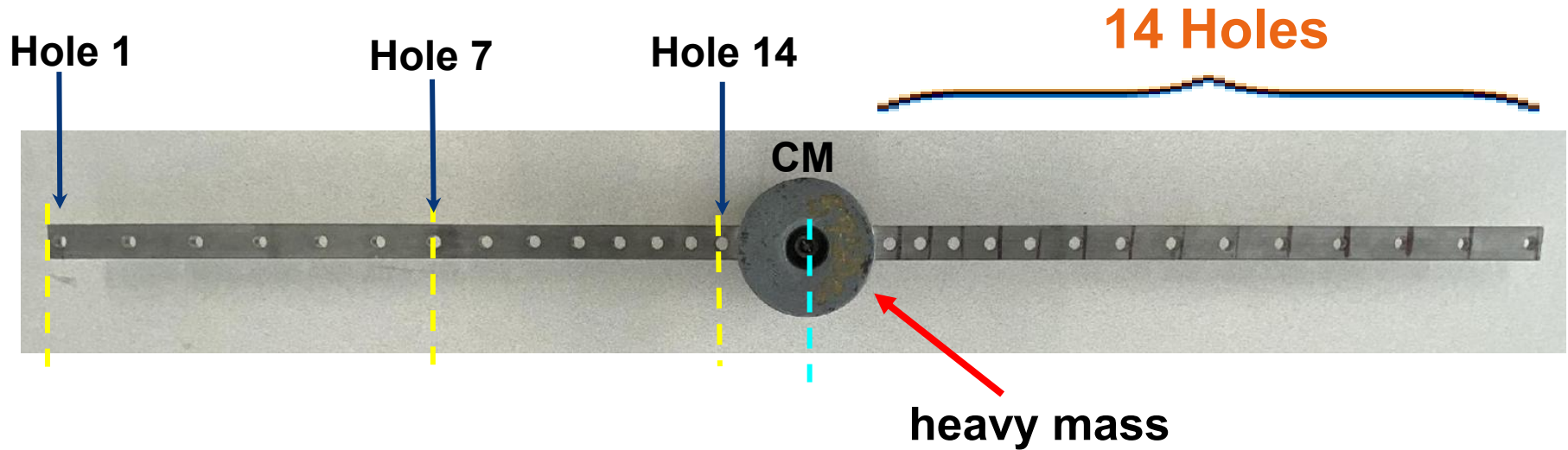
$$L = h_1 + h_2$$



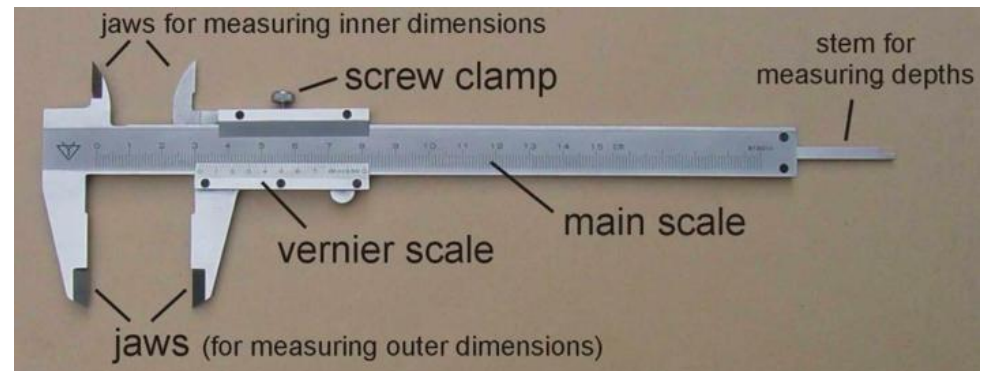
APPARATUS

THE PHYSICAL PENDULUM

Apparatus: Physical Pendulum, Vernier Calipers, Ruler, Stopwatch



**Vernier
Calipers**



EXPERIMENT

What to measure : Distance from one end to the center of the pendulum (D), mass of the pendulum (M), distance from one end to each suspension point (S), time for a given number of oscillations for each hole (t)

What to calculate : Period of oscillation (T), radius of gyration (k), moment of inertia (I)

Experimental findings : Gravitational acceleration (g)



THE PHYSICAL PENDULUM

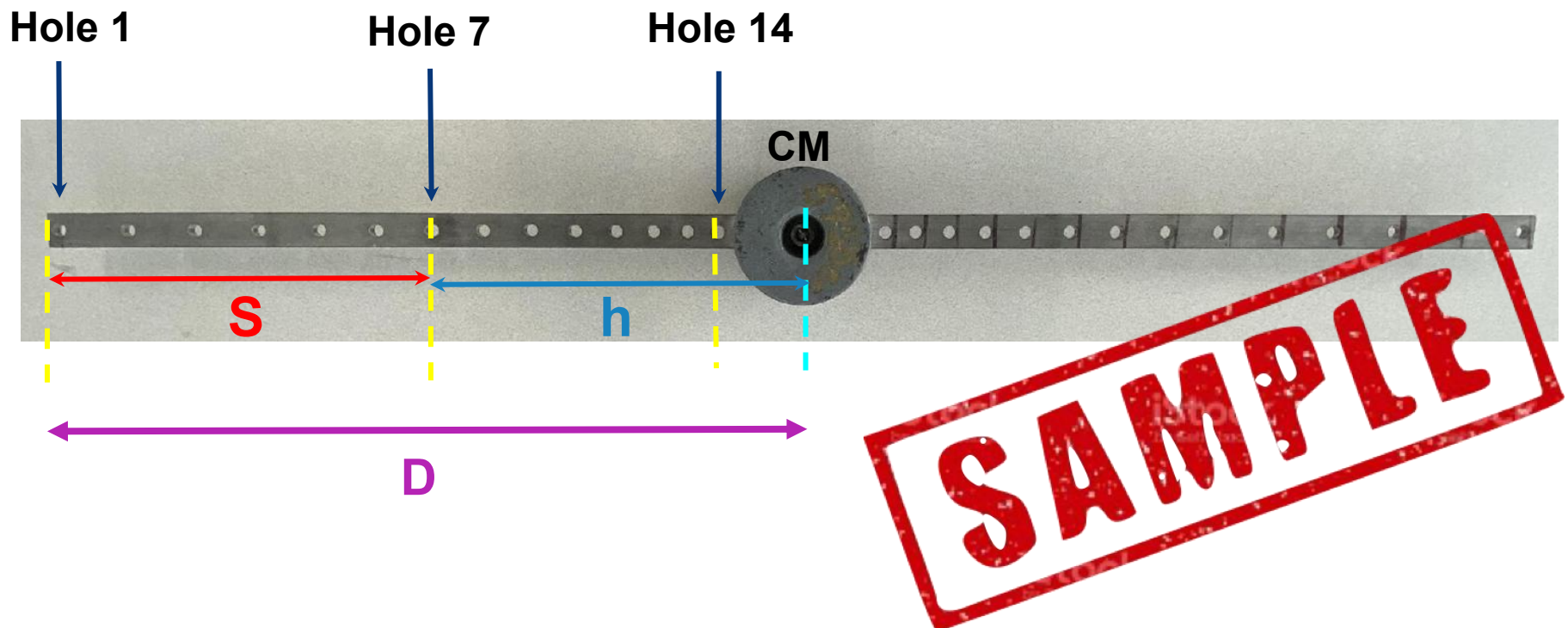
Distance from one end
to the center D = ...

Mass of M = ...

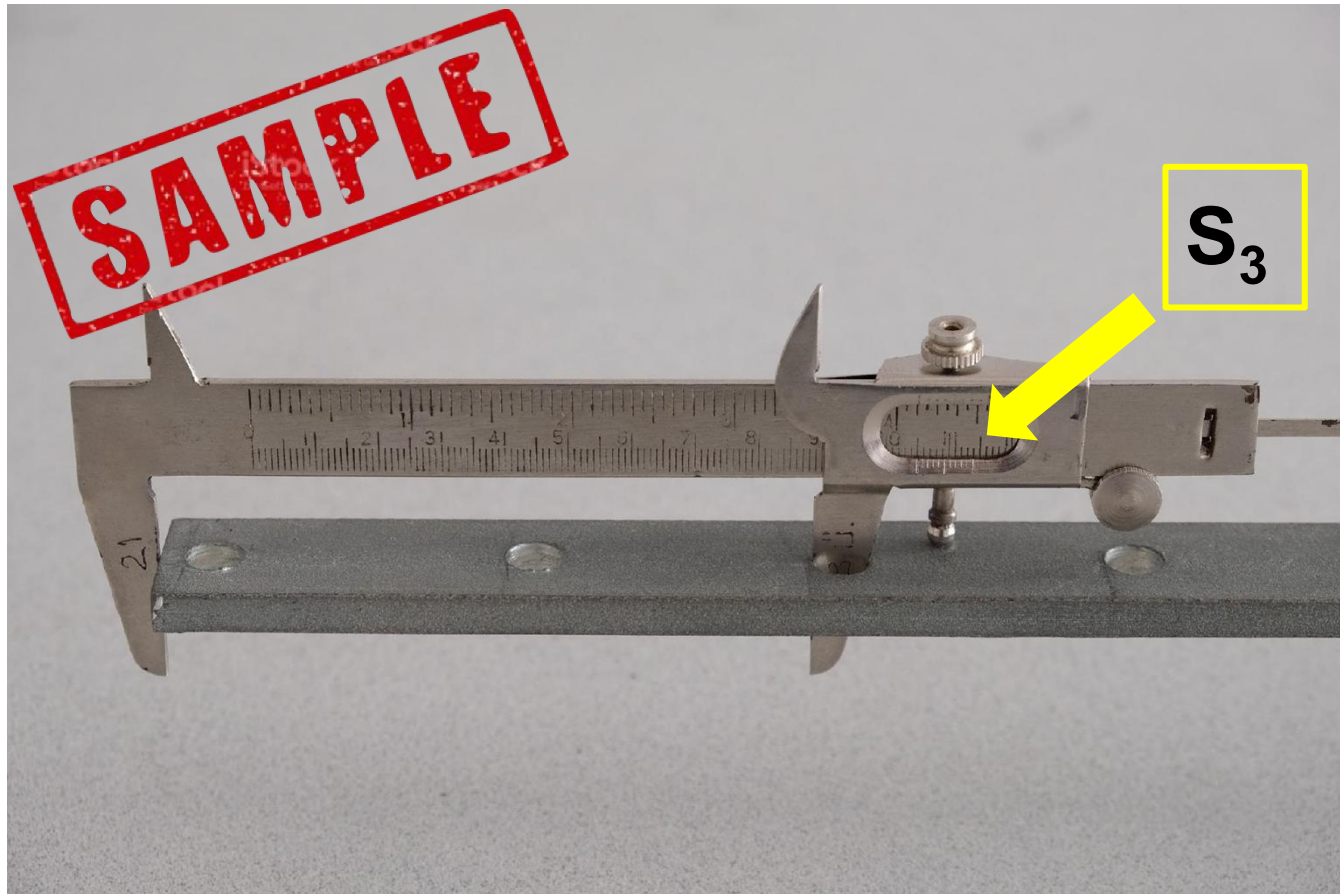
Acceleration
due to gravity g_{TV} = ...

PROCEDURE:

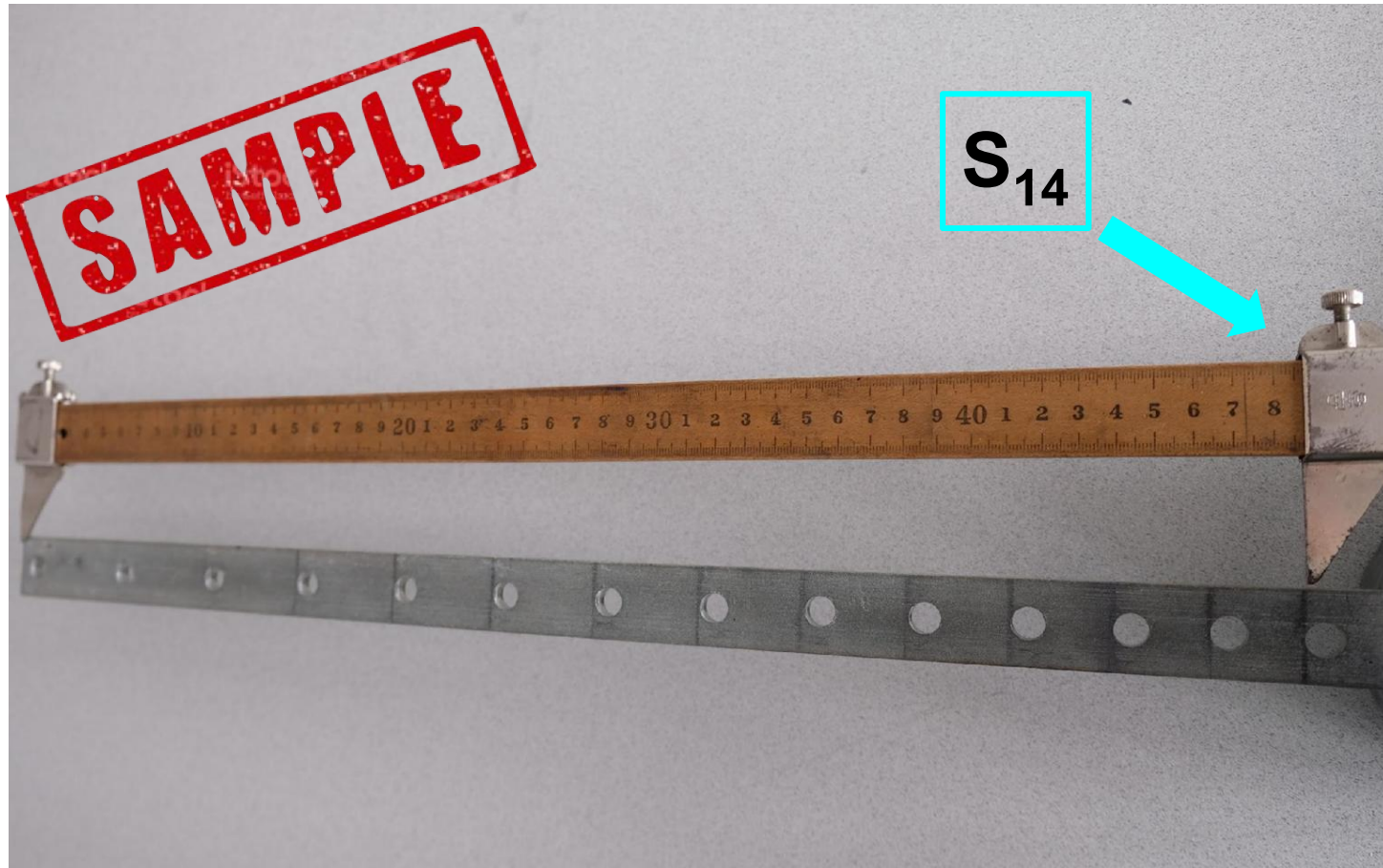
- ❑ Measure **Distance S** from one end (not the CM) to the suspension point for each hole.



- For SMALL distances (S_1 , S_2 , S_3), use Vernier Calipers

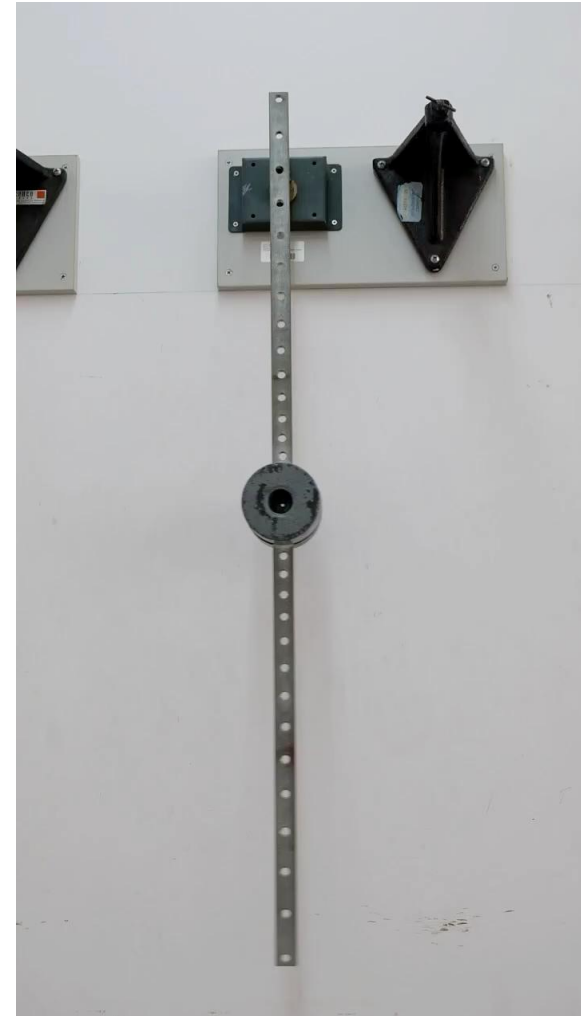


- For LARGER distances ($S_4, S_5, \dots, S_{14}$), use a Ruler



THE PHYSICAL PENDULUM

- ❑ Hang the physical pendulum from each hole and make it oscillate.
- ❑ Measure time t for the given number of oscillations for each hole using a stopwatch.



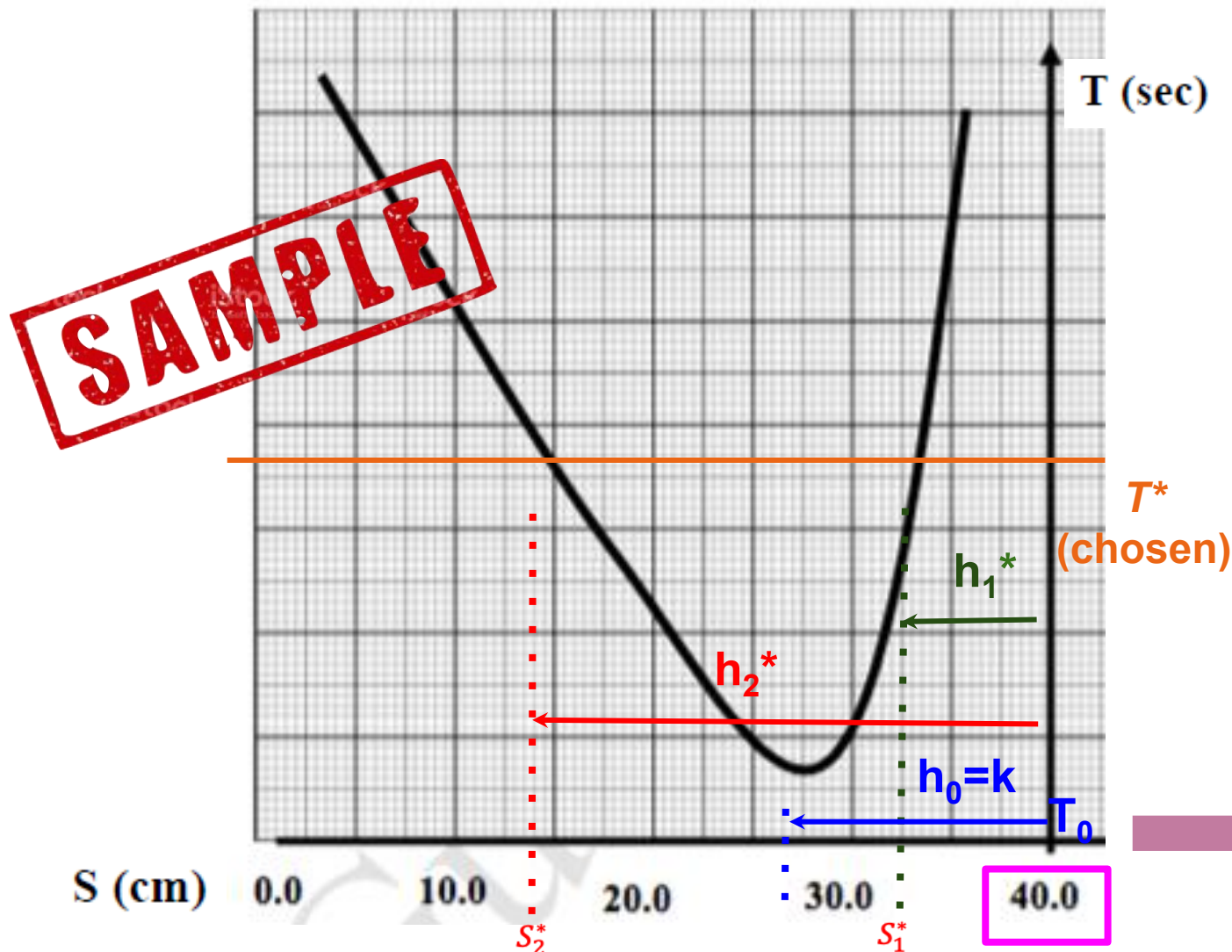
THE PHYSICAL PENDULUM



Distance from one end of the pendulum to the suspension point S ()	Time for 10 Period t ()	One Period T ()

THE PHYSICAL PENDULUM

- Plot S vs. T graph. Choose a period value T^* and draw a horizontal line corresponding to that period.



Place y-axis on
 $S=D$
point

THE PHYSICAL PENDULUM

READ VALUES FROM THE GRAPH YOU HAVE DRAWN.

Period (any chosen) T^* = ...

Minimum Period T_0 = ...

Distance from the center to the first suspension point for T , h_1 = ...

Distance from the center to the second suspension point for T , h_2 = ...

For minimum Period: h_0 = ...

Radius of Gyration $k = h_0$ = ...

***Determine h values
by subtracting
corresponding S^*
values on the
graph from
distance D !***

Calculate using h_1^* and h_2^* you have read from the graph!

Radius of Gyration $k = (h_1^* h_2^*)^{(1/2)} = \dots\dots\dots$

Length of the Equivalent
Simple Pendulum $L = h_1^* + h_2^* = \dots\dots\dots$

THE PHYSICAL PENDULUM

Moment of Inertia

about the CM $I_o = I_{CM} = \dots\dots\dots$

$\dots\dots\dots$

Moment of Inertia

Corresponding $I_{(\text{for } T)}$ to $h_1 = \dots\dots\dots$

$\dots\dots\dots$

Moment of Inertia

Corresponding $I_{(\text{for } T)}$ to $h_2 = \dots\dots\dots$

$\dots\dots\dots$

Acceleration
due to Gravity g_{EV}

$= \dots\dots\dots$

***Use length of
equivalent simple
pendulum L and the
chosen period T^* !***



% Error for g

=

.....

Dimensional analysis for the Radius of Gyration, k ..

Dimensional analysis for the moment of Inertia, I : ..

THE PHYSICAL PENDULUM



- BRING THE POST-LAB SHEET TO THE NEXT LAB SESSION.