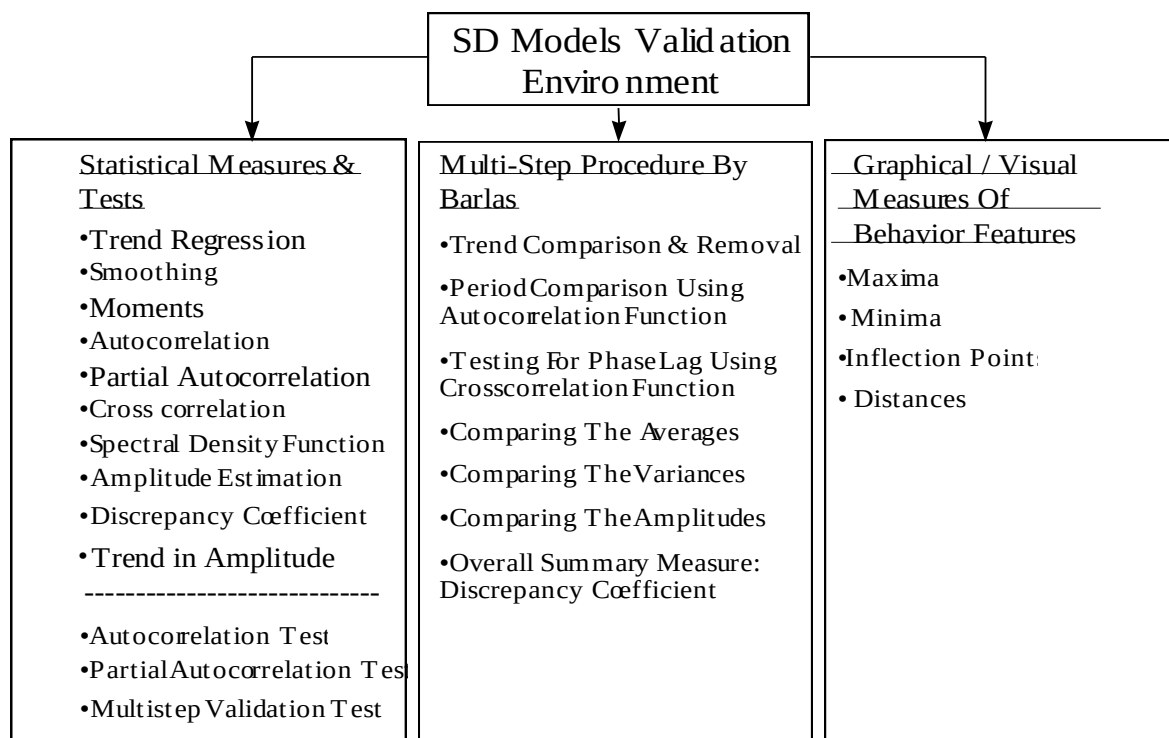


A BEHAVIOR VALIDITY SOFTWARE (BTS II)

In testing the behavior validity of a system dynamics model, the crucial task is to evaluate the pattern prediction ability. Thus, good models are expected to generate patterns (trends, periods, amplitudes, autocorrelations...) that match the ones that are observed in the real data.

The behavior validation environment has been developed to be used in Windows environment in order to provide users flexibility and the interaction of the environment with other Windows applications. Delphi is utilized in developing the environment. The environment is capable of using model output data of any simulation, interpreting STELLA II, VENSIM or POWERSIM tables that are saved as text. More than just applying the six step procedure developed by Barlas [1], the program can calculate various statistics individually. Also for highly non-stationary, transient behavior patterns, a graphical inspection interface has been embodied into the environment. The following table summarizes the main features of the environment.



The multi-step behavior pattern validation procedure was originally proposed by Barlas [1]. This procedure was later improved by Barlas et al (1997) to yield the behavior validity testing software BTS. In BTS II, we further improve BTS by modifying some of its existing tools and by adding new tools.

Trend Estimation and Removal

Trends are estimated by fitting regression curves to the outputs of the model and the actual system and the regression coefficients are compared. The regression models should be as simple as possible, because the objective is to estimate the trend rather than fitting a perfect curve. A higher order regression may show some maxima, minima, inflection points and oscillations which strictly oppose the definition of trend. The equations proposed by Barlas [1] are $\hat{Y} = b_0 + b_1t$, $\hat{Y} = b_0 + b_1t + b_2t^2$ and $\hat{Y} = b_0e^{bt}$. If the coefficients show no significant error, trend should be removed by $Z_i = Y_i - b_1t$, or $Y_i - b_1t - b_2t^2$ or $Y_i - b_0(e^{bt} - 1)$.

Autocorrelation Function (For Period Comparison)

Sample autocovariance and autocorrelation functions of a time series X_t is given by

$$Cov(k) = \frac{1}{N-k} \sum_{i=1}^{N-k} (x_i - \bar{x})(x_{i+k} - \bar{x}) \text{ for } k=0,1,2,\dots,N.$$

$$r(k) = \frac{Cov(k)}{Cov(0)} = \frac{Cov(k)}{Var(x)}$$

These functions give a measure of how the successive data points are interrelated. For a statistical test on the equality of the autocorrelation functions of the output of the model and the actual system, we need the sampling distribution of $r(k)$. As an approximation we can compute the standard errors (S_e) of the difference of $r(k)$ of the model and the actual system and construct a $2 \times S_e$ confidence band around the differences of the two autocorrelation functions. Anderson [5] provides the following $V(r(k))$ which is good enough for the purpose of the study.

$$Var(r(k)) = \frac{1}{N(N+2)} \sum_{i=1}^{N-k-1} (N-i)(r(k-i) + r(k+i) - 2r(k)r(i))^2$$

If $r_S(k)$ belongs to model output and $r_A(k)$ belongs to actual system output, then the statistical test becomes

$$H_0: r_S(1) - r_A(1) = 0, r_S(2) - r_A(2) = 0, \dots, r_S(n) - r_A(n) = 0$$

$$H_1: r_S(k) - r_A(k) \neq 0 \text{ for at least one } k.$$

$$d_k = r_S(k) - r_A(k)$$

$$Se(d_k) = \sqrt{Var(r_S(k)) + Var(r_A(k))}$$

H_0 is rejected if any d_k falls outside the confidence band $\pm 2 \times Se(d_k)$.

Estimating Multiple Periods by Spectral Density Function

In estimating multiple periods in data sets Spectral Density Function is used instead of Autocorrelation Function since multiple periods can be estimated with this function more clearly than with autocorrelation function.

Spectral Density Function is a continuous function [6]. Since time series obtained from system dynamics models and real systems are discrete points, an estimate of Spectral Density Function, Tukey-Hanning estimate [6] is used:

$$S(\omega_i) = 1/2\pi \sum_{j=1}^m \text{cov}(j) \cdot \lambda(j) \cdot \cos j\omega_i, \quad i = 1, \dots, m$$

where $S(\omega_i)$ is spectral density function, ω_i is frequency, $\lambda(j)$ is the weighting function, $\text{cov}(j)$ covariance at lag j and n is the length of the data set.

Parameters in the estimate function are chosen as follows:

$$m = n/2,$$

$$\omega_i = 2\pi / m, \quad i = 0, 1, \dots, m.,$$

$$\lambda(j) = 1 + \cos \pi j / m$$

Using Spectral Density Function estimate for each frequency value, ω_i , the spectral density function is constructed. Then the first and the second maximum among the function values, $S(\omega_p)$ and $S(\omega_q)$, are determined. These are the spectral density function values of the two

most dominant periods in dataset. Then these periods p and q are reported on the result sheet as dominant periods. A graphical representation of all spectral density function values is also provided.

Cross-Correlation Function (For Phase Lag Detection)

The maximum of the following cross-correlation function occurs at lag 0 if the output of the model and the actual system are perfectly in phase. Otherwise the function reaches its maximum value at some non-zero lag.

$$C_{SA}(k) = \frac{\frac{1}{N} \sum_{i=k+1}^N (S_i - \bar{S})(A_{i-k} - \bar{A})}{S_S S_A} \quad \text{for } k=0,1,2,\dots < N$$

$$C_{SA}(k) = \frac{\frac{1}{N} \sum_{i=-k+1}^N (A_i - \bar{A})(S_{i+k} - \bar{S})}{S_S S_A} \quad \text{for } k=0,-1,-2,\dots > -N$$

$$\text{where } \bar{S} = \frac{1}{N} \sum_{i=1}^N S_i, \quad \bar{A} = \frac{1}{N} \sum_{i=1}^N A_i, \quad s_S = \sqrt{\frac{1}{N} \sum_{i=1}^N (S_i - \bar{S})^2}, \quad s_A = \sqrt{\frac{1}{N} \sum_{i=1}^N (A_i - \bar{A})^2}.$$

Comparing the Means

If the output of the model and the actual system were not highly autocorrelated data sets, their means could be compared using a standard t-test. However as a practical approach,

percent error in means $E_1 = \frac{|\bar{S} - \bar{A}|}{|\bar{A}|}$ can be examined to see if there is a large discrepancy between the means.

Comparing the Variances

Percent error in the variations $E_2 = \frac{|s_S - s_A|}{|s_A|}$ can be examined to see if there is a large discrepancy between the variations.

Comparing the Amplitudes

Various extensions are made to Barlas's procedure for comparison of constant amplitudes and changing amplitudes:

Comparison of Constant Amplitudes

Three different approaches are provided in BTS II to estimate the constant amplitudes in datasets.

The **first approach** is to fit a trigonometric function to S_i & A_i , using regression. Regression approach yields a single amplitude estimate and a phase lag that defines the phase lag of the output behavior with respect to pure sine function. The core assumption is that sine function is a good approximation to the oscillatory behavior of the output.

The sine function, $A \times \sin\left(\frac{2\pi t}{T} + \Phi\right) + k$ is fitted to the output behavior where A is the amplitude, Φ is the phase lag and k is constant. With respect to parameters A and Φ , the regression model in question is not a linear model. In order to linearize the model, following procedure was employed.

$$A \times \sin\left(\frac{2\pi t}{T} + \Phi\right) = A \times \left(\sin\left(\frac{2\pi t}{T}\right) \cos(\Phi) + \cos\left(\frac{2\pi t}{T}\right) \sin(\Phi) \right)$$

Say $U_1 = A \times \cos(\Phi)$ and $U_2 = A \times \sin(\Phi)$. Then the model reduces to

$U_1 \sin\left(\frac{2\pi t}{T}\right) + U_2 \cos\left(\frac{2\pi t}{T}\right) + k$, which is linear in terms of parameters U_1 and U_2 . After estimating the coefficients U_1 and U_2 , back transformation gives the estimation of amplitudes and phase lag.

$$\frac{U_2}{U_1} = \frac{A \times \sin(\Phi)}{A \times \cos(\Phi)} = \tan(\Phi). \text{ Hence } \Phi = \text{ArcTan}\left(\frac{U_2}{U_1}\right) \text{ and } A = \frac{U_1}{\cos(\Phi)} = \frac{U_2}{\sin(\Phi)}.$$

The **second approach** is to use a modification of Winters seasonal forecasting procedure used in forecasting literature. The underlying model is $X_t = (a + bt) \times F_t + \varepsilon_t$. Since the trend is assumed to be removed beforehand, b is defined to be 0 in the following discussion. The initialization process of Winters' procedure is used to estimate amplitudes as described in [4].

Winters seasonal forecasting procedure works for only positive data and if any negative data exist, the minimum of the data should be subtracted from each data. The standard initialization process of Winters' seasonal forecasting procedure is carried on as given in [4]. A measure of amplitude is calculated by subtracting minimum of $(a \times \text{Normalized } F_t)$ from maximum of $(a \times \text{Normalized } F_t)$. The proposed methodology has been suggested by Barlas

and Erdem [5] but since they had negative values in their data sets, the usefulness of the procedure had been underestimated in their work.

In the **third approach** used to estimate the amplitude, dataset, $Y(i)$, is divided into subsets with length of T , where T is the period:

$$\begin{aligned} \text{subset1} &= \{Y(1), Y(2), Y(3), \dots, Y(T)\} \\ \text{subset2} &= \{Y(T+1), Y(T+2), Y(T+3), \dots, Y(2T)\} \\ &\vdots \\ \text{subset}n &= \{Y(nT+1), Y(nT+2), Y(nT+3), \dots\} \end{aligned}$$

where $n = 1 + \text{TRUNCATE}(\text{length of data set} / \text{period})$.

The amplitude of each subset is estimated separately using the sine function as in the first approach. Then the average of amplitude estimates of all subsets is accepted as the amplitude estimate of the dataset.

Estimating Trend in Amplitudes

Three different methods are used in order to estimate the trend in amplitudes.

For the **first and the second** method, dataset $Y(i)$, is divided into subsets with length of T , where T is the period:

$$\begin{aligned} \text{Subset-1} &= \{Y(0), Y(T), Y(2T), \dots, Y(nT)\} \\ \text{Subset-2} &= \{Y(1), Y(T+1), Y(2T+1), \dots, Y(nT+1)\} \\ &\vdots \\ \text{Subset-(T-1)} &= \{Y(T-1), Y(T+T-1), Y(2T+T-1), \dots\} \end{aligned}$$

where $n = \text{TRUNCATE}(\text{length of data set} / \text{period})$

Each subset is accepted as time series. Linear regression is applied to each of them and a slope is estimated from line obtained for each subset.

In the **first method**, the lines with highest positive and negative slopes pass through the tip points of amplitudes. Therefore in order to find the slope of amplitude trends, the lines with maximum and minimum slopes are selected. If it is believed that the behavior is asymmetric with respect to the x-axis, then this is the only method that can be used.

In the **second method**, data set is accepted as symmetric about the x-axis, so the average of absolute values of the maximum and minimum slope is taken. Thus a symmetric slope for upper and lower part of the behavior is obtained.

In the **third method**, dataset $Y(i)$, is divided into subsets with length of T , where T is the period:

$$\begin{aligned}\text{subset1} &= \{Y(1), Y(2), Y(3), \dots, Y(T)\} \\ \text{subset2} &= \{Y(T+1), Y(T+2), Y(T+3), \dots, Y(2T)\} \\ &\vdots \\ \text{subsetn} &= \{Y(nT+1), Y(nT+2), Y(nT+3), \dots\}\end{aligned}$$

where $n = 1 + \text{TRUNCATE}(\text{length of data set} / \text{period})$.

Amplitude of each subset is estimated by fitting trigonometric regression function and n number of amplitude estimations are obtained: Amp-1, Amp-2, ..., Amp-n. With the help of linear regression, these series of amplitudes are fitted onto a line whose slope will give the estimate of amplitude slope. The data set is again accepted to be symmetric about the x-axis.

Summary Measure of Overall Behavior Discrepancy

Single summary measures are not very informative in evaluating validity of model behavior. On the other hand single measures are simple vehicles of communicating the results. The following discrepancy coefficient should be interpreted with great caution and rather large values should be allowed.

$$U = \frac{\sqrt{\sum_{i=1}^N (S_i - \bar{S} - A_i + \bar{A})^2}}{\sqrt{\sum_{i=1}^N (A_i - \bar{A})^2} + \sqrt{\sum_{i=1}^N (S_i - \bar{S})^2}} = \frac{\sqrt{\sum_{i=1}^N (E_i - \bar{E})^2}}{\sqrt{\sum_{i=1}^N (A_i - \bar{A})^2} + \sqrt{\sum_{i=1}^N (S_i - \bar{S})^2}} = \frac{s_E}{s_A + s_S}$$

Smoothing

In order to improve the performance of tests and analysis performed in BTS II by reducing the noise level in datasets, some smoothing formulas can be initially applied to data. Two smoothing formulas exist in BTS II: Moving Average and Exponential Smoothing. The M -Point Moving Average formulation is simply:

$$\hat{Y}(t) = \{Y(t) + Y(t-1) + Y(t-2) + \dots + Y(t-M+1)\} / M$$

where $\hat{Y}(t)$ is smoothed level at time t , $Y(t)$ is the observed value at t and M is the moving average parameter.

The formula for Exponential Smoothing is as follows:

$$\hat{Y}(t) = \alpha \cdot \hat{Y}(t-1) + (1-\alpha)Y(t), \quad 0 < \alpha < 1$$

where $\hat{Y}(t)$ is smoothed level at time t , $Y(t)$ is the observed value at t and α is smoothing constant.

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